longer wavelength perturbations, and then the shortwave perturbations. The latter is in qualitative agreement with the data of an experimental investigation of the influence of the bluntness of a body leading edge on the stability of boundary layer flow with an external supersonic stream. The state in investigations of perturbation development processes at supersonic velocities, and in particular, the role of body leading edge bluntness in the loss of flow stability in the boundary layer, is examined in detail in [9].

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HEAT AND MOMENTUM TRANSFER IN A TURBULENT BOUNDARY LAYER ON A CURVED SURFACE

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It is known [1-11] that the presence of relatively small streamwise curvature can have significant effect on turbulent heat and mass transfer and skin friction. Here the consideration of only the deformation of boundary layer, characterized by the ratio of boundary layer thickness to radius of curvature δ/R, leads to an appreciably lower effect of curvature on skin friction and heat transfer [2, 12] when compared to experiment. Prandtl [1] was one of the first to show the similarity between the effects of buoyant forces in stratified fluid and streamline curvature in boundary layer. He used mixing length hypothesis to suggest the following relation for turbulent skin friction: $\tau/\tau_0 = \sqrt{1 - 0.5\mathrm{Ri}}$. The Richardson number used here as the parameter differed from its usual form for stratified fluid by the replacement of acceleration of gravity by centripetal acceleration. However experimental verification of Prandtl's hypotheses showed [8] that the observed effect is an order higher than that given by theory. Empirical relations between mixing length and boundary layer parameters and streamline curvature were used to study this problem [2-7]. The basis for these methods is the analysis of Monin and Obukhov for the computation of temperature-stratified atmospheric boundary layers. Thus, it was suggested in [2] to use different relations for modified mixing length, in particular a linear relation

$$l/l_0 = 1 - \beta\mathrm{Ri}, \tag{0.1}$$

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where l, l_0 are the mixing lengths for curvilinear and plane boundary layers; the constant β is chosen empirically. Computational methods using these relations give satisfactory agreement with experiment even in the case of more complex flows, e.g., curved flow under nonisothermal conditions [11]. However, the question of the proper choice of empirical coefficient β remains extremely complex and not resolved to the present time. A method to compute turbulent momentum, heat, and mass transfer in curvilinear boundary layers has been described in this paper without the use of empirical constants that depend on streamline curvature.

1. Influence of Body Forces on Turbulence

It is known that turbulent boundary layer can be conditionally split into two parts: the wall layer including laminar sublayer and the buffer region, and the outer region of the boundary layer. The wall region of the boundary layer is characterized by the transport of small eddies from the laminar sublayer to the outer region and these transports are random in nature with a length scale of the order of the distance from the wall [13]. The outer region of the boundary layer is characterized by the presence of weakened eddies transported from the wall region and these eddies do not have appreciable influence on skin friction in the wall region. Apparently, turbulence in the outer region of the boundary layer basically affects skin friction in the wall region by altering the momentum transfer coefficients and as a result of a fuller velocity profile in the outer region of the boundary layer. Unlike turbulent shear stress which is characterized by eddy size of the order of mixing length, the maximum effect of body forces on turbulence is through large eddies with length scale of the order of boundary layer thickness. In the subsequent breakdown of the largest eddies, small eddies in their turn get energy from larger eddies. As shown by computations described below, body forces affect turbulent transport only in the outer region of the boundary layer.

Let us analyze the effect of body forces on the dynamics of a gram-molecule of fluid as it moves from the wall to the outer edge of the boundary layer. The equation of motion of the fluid particle in the radial direction at the concave surface (in Lagrangian coordinate system) is written in the form:

$$\int_V \rho_s \frac{dv_r'}{dt} dV = \int_V f_m dV - \left(\int_s p ds\right)_r. \tag{1.1}$$

Here v_r' is the fluid particle velocity fluctuation; ρ_s is its density; f_m are body forces acting on the gram-molecule in the radial direction. The last term in (1.1) represents pressure forces. The index s denotes the parameter of the fluid particle displaced from the point $r + y$ to the point r; $y = (R - r)$ is the distance from the wall.

In writing (1.1) it was assumed that shear stresses acting on the particle are appreciably less than pressure and inertia forces. This is because viscous dissipation of energy takes place in eddies whose size is considerably smaller than those responsible for momentum transport.

Equation (1.1) can be written in the form

$$\int_V \left(\rho_s \frac{dv_r'}{dt} - f_m + \frac{dp}{dr}\right) dV = 0.$$

Since the volume V is arbitrary,

$$\rho_s dv_r'/dt = f_m - dp/dr. \tag{1.2}$$

Body force f_m acting on the fluid particle is made up of centrifugal force $f_c = \rho_s u_s^2/r$ and the force f_0 that does not depend on curvature and leads to the appearance of velocity fluctuations in the boundary layer in the absence of body forces:

$$f_m = \rho_s u_s^2/r - f_0. \tag{1.3}$$

The pressure gradient in radial direction can be found from the equation

$$dp/dr = \rho u^2/r. \tag{1.4}$$

Taking into consideration Eqs. (1.3) and (1.4), Eq. (1.2) takes the form

$$\rho_s v_r' \frac{dv_r'}{dr} = \rho_s \frac{u_s^2}{r} - \rho \frac{u^2}{r} - f_0. \tag{1.5}$$

We assume that the fluid mole conserves its momentum as it moves across the boundary layer. In view of this, the quantity $(u^2r^2)_{r+y}$ is represented in series, and limited to only linear terms in the first approximation:

$$(u^2r^2)_{r+y} = u^2r^2 + y\partial u^2r^2/\partial r. \tag{1.6}$$

Similar linear dependence can be obtained from relations used in Prandtl's mixing length hypothesis for the region where wall law is valid.

The expansion (1.6) is used to obtain an expression for the resultant force acting on the fluid particle during its transport:

$$\begin{gathered}\frac{\rho_s u_s^2}{r} - \rho\frac{u^2}{r} = \rho_s\left[\frac{(u^2r^2)_{r+y}}{r^3} - \frac{\rho u^2}{\rho_{r+y}r}\right] \approx \\ \approx \rho_s\left(\frac{u^2r^2 + 2ur\frac{\partial ur}{\partial r}y}{r^3} - \frac{\rho}{\rho + \frac{\partial\rho}{\partial r}y}\frac{u^2}{r}\right) \approx \rho_s\left(\frac{2u}{r^2}\frac{\partial ur}{\partial r} + \frac{1}{\rho}\frac{\partial\rho}{\partial r}\frac{u^2}{r}\right)y.\end{gathered} \tag{1.7}$$

Substituting (1.7) in (1.5), we obtain

$$dv_r'^2/dr = 2Ky - 2f_0/\rho_s.$$

Here

$$K = 2\frac{u}{r^2}\frac{\partial ur}{\partial r} + \frac{1}{\rho}\frac{\partial\rho}{\partial r}\frac{u^2}{r}. \tag{1.8}$$

Or, switching to the coordinates $y = R - r$

$$\partial v_r'^2/\partial y = -2Ky + 2f_0/\rho_s. \tag{1.9}$$

The velocity fluctuation in radial direction is determined by integrating (1.9) and assuming that the integration limit coincides with the distance from the wall:

$$v_r'^2 = -Ky^2 + \int_0^y \frac{2f_0}{\rho_s}dy. \tag{1.10}$$

The equation (1.10) is obtained for a constant value of the parameter K(r) = const, though, as it follows from (1.8), the quantity K is determined from the velocity and density distribution in the boundary layer. However, specially carried out computations showed that the use of exponential velocity profile in Eq. (1.8) and the subsequent integration (1.9) for the variable across the boundary layer K leads to values of velocity fluctuation close to those obtained from Eq. (1.10). In the absence of body forces ($K \to 0$) the velocity fluctuation is obtained according to Eq. (1.10) as

$$v_{r0}'^2 = \int_0^y \frac{2f_0}{\rho_s}dy, \tag{1.11}$$

where $v_{ro}'^2$ is the square of the velocity fluctuation in the absence of body forces on curvilinear surface.

Using Eq. (1.11), Eq. (1.10) is expressed in the form

$$v_r'^2 - v_{r0}'^2 = -Ky^2. \tag{1.12}$$

Velocity fluctuation v_{ro}' in the absence of body forces is expressed in the form normally used in mixing length hypothesis [1]:

$$v_{r0}' = l_0\left(\frac{1}{r}\frac{\partial ur}{\partial r}\right).$$

Then from (1.12)

$$v_r' = l_0\frac{1}{r}\frac{\partial ur}{\partial r}\sqrt{1 - (y/l_0)^2\,\mathrm{Ri}}, \tag{1.13}$$

where the Richardson number has the form

$$\mathrm{Ri}=\frac{K}{\left(\frac{1}{r}\frac{\partial ur}{\partial r}\right)^2}=\frac{2\frac{u}{r^2}\frac{\partial ur}{\partial r}+\frac{1}{\rho}\frac{\partial \rho}{\partial r}\frac{u^2}{r}}{\left(\frac{1}{r}\frac{\partial ur}{\partial r}\right)^2}. \tag{1.14}$$

Richardson number (1.14) determines the ratio of turbulent energy production by body forces to energy production by shear stresses. Here body forces can be not only due to circulation gradient but also density gradient in the boundary layer. For boundary layer on curved surface with constant density (ρ = const) Eq. (1.14) coincides with the familiar form of Richardson number [1]:

$$\mathrm{Ri}=\frac{(2u/r)}{\left(\frac{1}{r}\frac{\partial ur}{\partial r}\right)}.$$

The shear stress distribution across the boundary layer is obtained using Eq. (1.13), taking into consideration $u' = l_0(1/r)\partial ur/\partial r$ [1],

$$\tau=\rho u'v'=\rho u'v_0'\sqrt{1-(y/l_0)^2\,\mathrm{Ri}}=\rho l_0^2\left(\frac{1}{r}\frac{\partial ur}{\partial r}\right)^2\sqrt{1-(y/l_0)^2\,\mathrm{Ri}}. \tag{1.15}$$

Equation (1.9) is valid only for flow along concave wall. For the flow near convex surface turbulence is attenuated and the scale of fluctuations becomes less than boundary layer thickness.

Velocity fluctuation for convex surface is obtained by integrating equation of motion (1.9) over certain linear scale H

$$v_r'^2=-KH^2+v_{r0}'^2. \tag{1.16}$$

It follows from (1.16) that at a certain limiting distance $H = v'_{r0}/\sqrt{K}$ velocity fluctuation becomes zero which corresponds to conditions for turbulence attenuation.

The attenuation scale can be determined from the following obvious limiting relations: if $\mathrm{Ri} \to 0$, then $H \to Y$, $KH^2 \to 0$, if $\mathrm{Ri} \to \infty$, then $H \to 0$, $v_r'^2 \to 0$. The function $H = y/\sqrt{1 + (y/l_0)^2\mathrm{Ri}}$.

Equation (1.16) takes the following form using this relation

$$v_r'=l_0\frac{1}{r}\frac{\partial ur}{\partial r}\Big/\sqrt{1+\left(\frac{y}{l_0}\right)^2\mathrm{Ri}}.$$

The distribution of turbulent shear stresses across boundary layer thickness on a convex surface is related to mean flow parameters by the expression

$$\tau=\rho\overline{u'v'}=\rho l_0^2\left(\frac{1}{r}\frac{\partial ur}{\partial r}\right)^2\Big/\sqrt{1+\left(\frac{y}{l_0}\right)^2\mathrm{Ri}}. \tag{1.17}$$

Similar analysis can be carried out even for turbulent heat transfer. Finally we obtain heat flux distribution near concave wall

$$q=c_p\rho\overline{T'v_r'}=c_p\rho l_0^2\frac{\partial T}{\partial r}\left(\frac{1}{r}\frac{\partial ur}{\partial r}\right)\Big/\sqrt{1-\left(\frac{y}{l_0}\right)^2\mathrm{Ri}} \tag{1.18}$$

and correspondingly near convex wall

$$q=c_p\rho\overline{T'v_r'}=c_p\rho l_0^2\frac{\partial T}{\partial r}\left(\frac{1}{r}\frac{\partial ur}{\partial r}\right)\Big/\sqrt{1+\left(\frac{y}{l_0}\right)^2\mathrm{Ri}}. \tag{1.19}$$

Thus, using Eqs. (1.15), (1.17)-(1.19) it is possible to take into account the effect of curvature on skin friction and heat and mass transfer for turbulent flow along concave and convex surfaces. Here it is not necessary to use additional constants that take into account the effect of body forces on turbulent transport.

2. Law of Heat and Mass Transfer and Skin Friction for Turbulent Boundary Layer on Curved Surface

Asymptotic theory of turbulent boundary layer [14] is used to compute the law of heat and mass transfer and skin friction. In accordance with this theory and using expressions for turbulent skin friction on curved surface (1.15) or (1.17), the relative skin friction function is found

$$\Psi = \left(\int_0^1 \left(\frac{\rho \tilde{\tau}_0 f}{\rho_0 \tilde{\tau}} \right)^{1/2} \frac{1 \mp \frac{\delta}{R}}{\left(1 \mp \xi \frac{\delta}{R}\right)} d\omega \right)^2, \tag{2.1}$$

where $\Psi = (c_f/c_{f0})_{\mathrm{Re}^{**}}$ is the relative skin friction function for Re** = idem; $c_f = 2\tau_w/\rho u_0^2$, $c_{f0} = 2\tau_{w0}/\rho u_0^2$ are skin friction coefficients under the present and standard (isothermal, plane impermeable surface) conditions; $\tilde{\tau}_0 = (\tau/\tau_w)_0$, $\tilde{\tau} = (\tau/\tau_w)$ are shear stress profiles under standard and present conditions; $\omega = ur/u_0(R - \delta)$ is nondimensional circulation; f is the function that determines the effect of body forces on skin friction according to (1.15) and (1.17): For concave surface $f = \sqrt{1 - (y/l_0)^2 \mathrm{Ri}}$, for convex surface $f = 1/\sqrt{1 + (y/l_0)^2 \mathrm{Ri}}$.

Relative shear stress distribution in the boundary layer on curved surface can be written according to [15] in the form

$$\frac{\tilde{\tau}}{\tilde{\tau}_0} = 1 \pm \frac{\delta}{R} \frac{2\xi}{1 + 2\xi}. \tag{2.2}$$

Here and in what follows the upper sign before δ/R corresponds to flow past concave wall; distribution of relative shear stresses in standard boundary layer is $\tilde{\tau}_0 = 1 - 3\xi^2 + 2\xi^3$.

Since the parameter f is a function of the distance from the wall, then in order to determine skin friction function the limiting relations (2.1) for isothermal conditions are transformed to the form

$$\Psi = \left(\int_0^1 \left(\frac{\tilde{\tau}}{\tilde{\tau}_0 f} \right)^{1/2} \frac{\left(1 \mp \frac{\delta}{R} \xi\right)}{\left(1 \mp \frac{\delta}{R}\right)} d\omega_0 \right)^{-2}, \tag{2.3}$$

where $\xi = y/\delta$; ω_0 is the velocity profile in standard boundary layer.

Circulation distribution in boundary layer on curved surface can be determined from the relation

$$\frac{\partial \omega}{\partial \xi} = \frac{\partial \omega_0}{\partial \xi} \left(\frac{\Psi \tilde{\tau}}{f \tilde{\tau}_0} \right)^{1/2} \frac{\left(1 \mp \frac{\delta}{R} \xi\right)}{\left(1 \mp \frac{\delta}{R}\right)}, \tag{2.4}$$

or after integration

$$\omega = 1 - \int_\xi^1 \frac{\partial \omega_0}{\partial \xi} \left(\frac{\Psi \tilde{\tau}}{f \tilde{\tau}_0} \right)^{1/2} \frac{\left(1 \mp \frac{\delta}{R} \xi\right)}{\left(1 \mp \frac{\delta}{R}\right)} d\xi. \tag{2.5}$$

It is possible to use Eq. (2.5) to compute integral relations of the boundary layer:

$$\delta^{**}/\delta = \int_0^1 \omega (1 - \omega) \frac{\left(1 \mp \frac{\delta}{R}\right)^2}{\left(1 \mp \xi \frac{\delta}{R}\right)} d\xi. \tag{2.6}$$

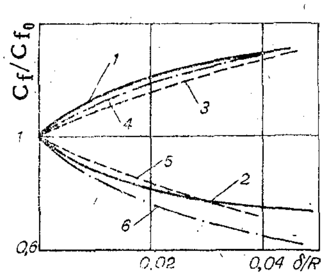

Fig. 1

The system of equations (2.3)-(2.6) makes it possible to determine all the required boundary layer parameters on curved surface. It is necessary to carry out computations by the method under standard conditions an exponential velocity profile $w_o = \xi^n$ was chosen.

The distribution of mixing length across the boundary layer on a plane surface l_o was computed using the equation

$$l_0/\delta = 0.4\xi - 0.64\xi^2 + 0.44\xi^3 - 0.11\xi^4,$$

obtained by approximating the expression for $\tilde{l} = l_o/\delta$ by a biquadratic polynomial using the following boundary conditions:

$$\xi = 0 \rightarrow \partial l_0/\partial\xi = \varkappa,\ l_0 = 0;\ \xi = 1 \rightarrow l_0 = 0.09,\ \partial l_0/\partial\xi = 0.$$

Velocity profile for the flow without curvature was used as the first approximation to determine Richardson number and corresponding skin friction function from Eq. (2.3).

The problem of heat and mass transfer could be solved in a similar manner. Since the expressions for turbulent heat transport (1.18) and (1.19) were similar to the corresponding relations for turbulent skin friction (1.15) and (1.17) on concave and convex surfaces, then if hydrodynamic and heat transfer mixing lengths are equal, $l_o = l_{oT}$, the heat transfer function will coincide with skin friction function. Thus, the above-described computational method for skin friction on curved surface using Eqs. (2.3)-(2.6) can also be used to compute heat and mass transfer processes.

Lines 1 and 2 in Fig. 1 represent computed results for skin friction on concave and convex walls respectively. The data are given in the form of expressions for relative skin friction coefficient c_f/c_{fo} as a function of the ratio of boundary layer thickness to the radius of curvature δ/R, which, as shown by analysis [2], determines quite well the effect of curvature-induced body forces on turbulence, i.e., the parameter δ/R in this case is the integral analog of Richardson number. Neglecting the geometric effect of curvature in Eqs. (2.3)-(2.6) results in a difference of less than 1% from the data given in Fig. 1 where results are based on computation of skin friction using empirical relations suggested by different authors for modified mixing length.

A good agreement with experiment is seen for the linear relation (0.1) when the coefficient $\beta = 2$ (line 3) for the flow past concave surface. The line 4 represents the relation using the equation for mixing length $l/l_o = \sqrt[4]{1 - 18\ \mathrm{Ri}}$ which, as shown in [2], gives more satisfactory results in the computation of skin friction on concave surface for large values of δ/R compared to values from formula (0.1).

For flow past convex surfaces, according to [2], it is necessary to put $\beta = 3$ in the formula (0.1), and according to [5, 7], $\beta = 6$. The line 5 in Fig. 1 represents computed results for the mean value of the coefficient $\beta = 4$. Here it is necessary to keep in view that in the outer region of the boundary layer the formula (0.1) gives physically incorrect result. If the value of the velocity gradient in the outer region of the flow becomes less than the ratio $\partial u/\partial y \leqslant 0.3u/\delta$, then it is necessary to determine $\partial u/\partial y$ from the relation [7] $\partial u/\partial y = 0.3u/\delta$. The condition was taken into consideration in computing the curve 5 in Fig. 1.

Apart from the above-described methods to represent modified mixing length, expressions of the following type [2, 6] are also used for flow past convex surfaces

$$l/l_0 = 1/(1 + \beta\ \mathrm{Ri}). \tag{2.7}$$

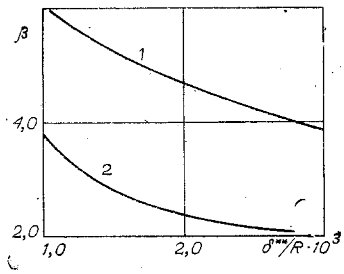

Fig. 2

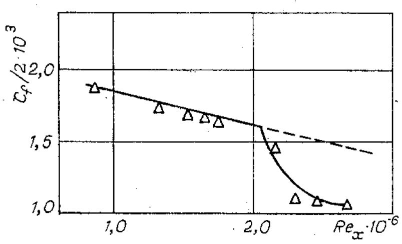

Fig. 3

The curve 6 in Fig. 1 represents computations using Eq. (2.7) and $\beta = 6$.

As seen from Fig. 1, computational results for skin friction using the present model for convex and concave surfaces agree well with known methods using different empirical relations for modified mixing length.

Figure 2 shows theoretically determined values of the coefficient β for convex (curve 1) and concave (curve 2) surfaces as a function of the parameter δ^{**}/R, which is more convenient to use in practical computations. It is shown that the value of β varies in the range given in [2, 5, 7], where the coefficient β is not a constant and decreases with an increase in curvature δ^{**}/R.

3. Solution of Momentum Integral Relations on Curved Surface. Comparison with Experiment

In order to solve the problem we use boundary layer integral relations. Momentum integral relations for curved boundary layer can be written in the form

$$d\delta^{**}/dx = c_f/2, \tag{3.1}$$

where δ^{**} is the momentum thickness given by Eq. (2.6).

Using the power law for skin friction on a flat plate,

$$c_f/2 = (B/2)\,\mathrm{Re}^{**-m}\,\Psi. \tag{3.2}$$

Here $\mathrm{Re}^{**} = \rho_0 u_0 \delta^{**}/\mu$; $B/2 = 0.0128$; $m = 0.25$, and the skin friction function is determined by using Eq. (2.3).

The integration of Eq. (3.1) using Eq. (3.2) gives computational relations for Reynolds number based on momentum thickness and local skin friction coefficient:

$$\mathrm{Re}^{**} = 0.036\Psi^{0.8}\,\mathrm{Re}_x^{0.8}; \tag{3.3}$$

$$c_f/2 = 0.029\Psi^{0.8}\,\mathrm{Re}_x^{-0.2}. \tag{3.4}$$

Equations (3.3) and (3.4) are obtained with the condition that the turbulent boundary layer grows from the leading edge of the curved channel $x = 0$, $\mathrm{Re}^{**} = 0$.

Thus, the system of equations (2.3)-(2.6), (3.3), and (3.4) makes it possible to compute completely the distribution of skin friction, velocity profiles, and integral characteristics along the curved channel. The solution of integral relations for energy and diffusion and the determination of corresponding characteristics of the thermal and diffusive layer do not present any significant difficulties.

Extensive experimental materials have accumulated up to the present time on the dynamics and heat transport in curved channels. However an analysis of these results showed that the absence of data in the plane pre-insert region, the presence of pressure gradient along the curved channel, etc. make it difficult to compare correctly theory with experiment. In view of this, data from [3] on skin friction along convex surface are of interest; in these experiments special steps were taken to eliminate streamwise pressure gradient.

A comparison of experimental data and computations based on present method is shown in Fig. 3. The effect of the presence of pre-insert plane segment of the channel on skin friction in the curved channel was taken into consideration using effective radius of curvature of the wall from the relation [3]

$$\frac{d\left(\frac{1}{R_{ef}}\right)}{dx}=\frac{1}{10\delta}\left(\frac{1}{R}-\frac{1}{R_{ef}}\right), \tag{3.5}$$

where R_{ef} is the effective radius of curvature; R is the geometric radius of curvature of the wall.

For small distances from the leading edge of the curved segment, Eq. (3.5) can be approximately expressed in the form

$$1/R_{ef} = (1/R)[1 - \exp(-s/10\delta)],$$

where s is the distance from the leading edge of the curved segment. The leading edge of the curved channel in Fig. 3 corresponds to $Re_x \approx 2 \cdot 10^6$. It follows from the data in Fig. 3 that computed results agree well with experiment, which confirms the correctness of the hypothesis used in modeling the transport.

Similar approach could be used to analyze a wide range of problems in boundary layers with body forces.

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